Cardiac MRI Segmentation

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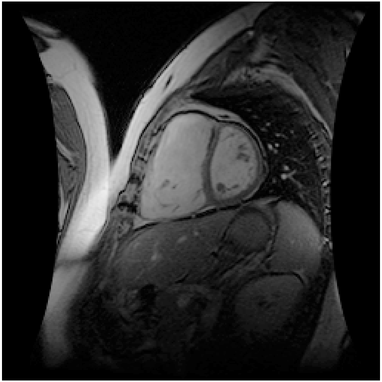
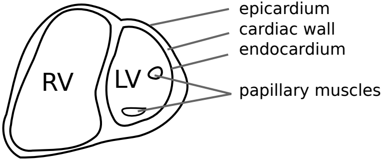
Outline

1. Problem description
2. Model description
3. Segmentation / inference
4. Parameter estimation
5. Results

Magnetic Resonance Image of cardiac structure

Short axis view of two heart ventricles ("chambers")

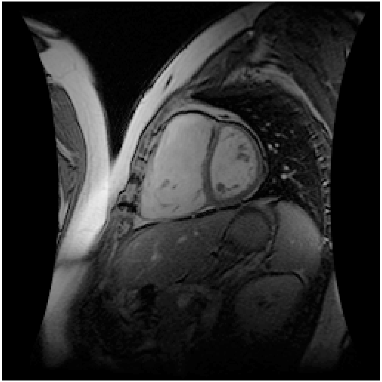


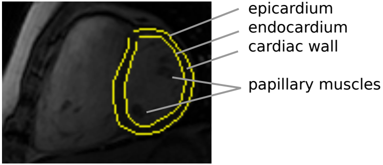


- MRI intensity corresponds with water content (e.g. blood pool)
- cardiac wall (muscle) smaller intensity values

Interested primarily in annotating the LV

Inner / outer contours

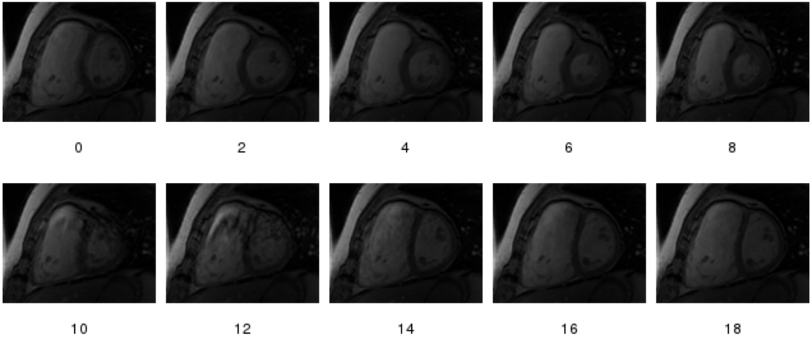




- LV does the hard work
- LV anno useful to calculate diagnostically important properties such as volume, ejection fraction

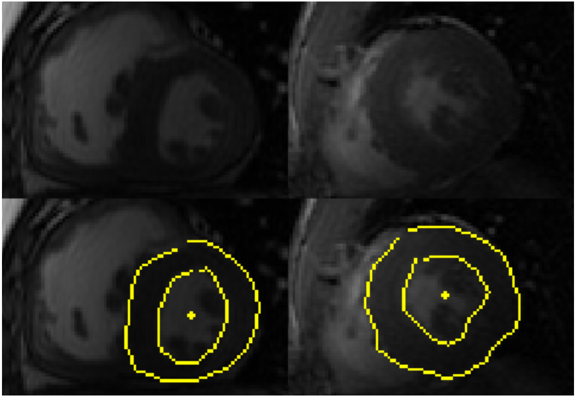
Temporal Sequence

Annotation is time consuming 20x10 frames per video



Annotation is not so simple

Papillary muscles obscure edge



Related work

Existing techniques

- AAM (3D+time)
- surface modelling MRFs
- snakes
- shortest path

Papillary muscles causes problems

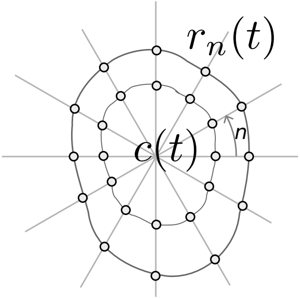
- area inside not homogeneous, edges obscured
- inner contour can disappear or be almost completely obscured in lower slice / max contraction
- global shape models trained on healthy hearts

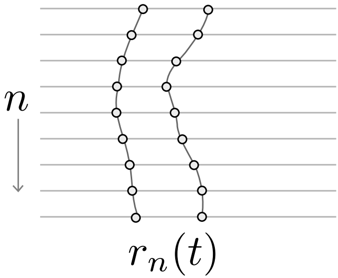
Our approach

- Integrate features describing local shape, appearance and temporal behaviour into a log-linear CRF

Polar transform

Inner / outer = series of radii around a centre point, circle becomes line





$$\rho_n(t) = \lfloor M \cdot r_{\text{init}} \cdot \log r_n(t) \rfloor . \tag{1}$$

Centre point estimation

Sequence of frames => sequence of centre points

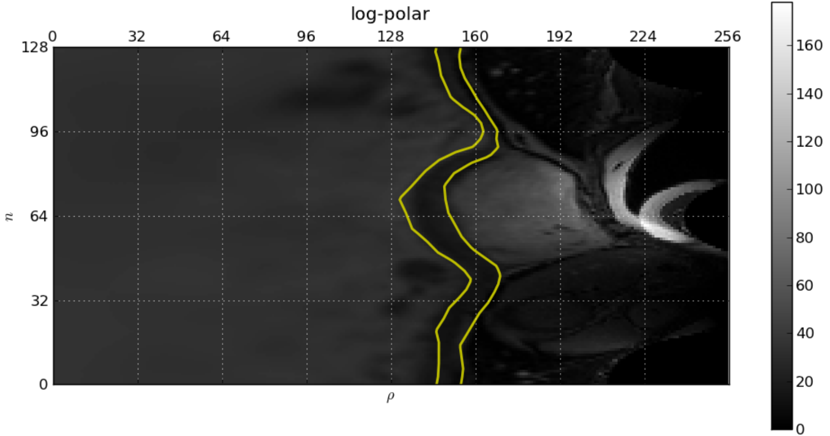
- We do offline tracking
- Given initial frame $\mathbf{c}(0)$, assume periodic heart cycle $\boldsymbol{c}(T-1) = \boldsymbol{c}(0)$
- We want to derive centre points for intermediate frames: $\boldsymbol{c}(1), \ldots, \boldsymbol{c}(T-2)$
- Minimise weighted inter-frame alignment error

$$\text{error}(\boldsymbol{c}) = \sum_{t=1}^{T-1} \sum_{\boldsymbol{p}} w^{\boldsymbol{c}(t)}(\boldsymbol{p}) \cdot \left(I^{\boldsymbol{c}(t)}(t, \boldsymbol{p}) - I^{\boldsymbol{c}(t-1)}(t-1, \boldsymbol{p}) \right)^2 \quad (2)$$

- $w^{\boldsymbol{c}(t)}(\boldsymbol{p}) = e^{\left(-\|\boldsymbol{c}(t)-\boldsymbol{p}\|^2/\sigma^2\right)}$ locally enhances the error around the current frame's centre point.
- Efficiently solved with dynamic programming

Log polar transform

small radii, wall thickness becomes scale invariant



Log polar formulation of segmentations

To summarize, for a single frame

$$\rho^{\text{in}}(t) = \left\{\rho_0^{\text{in}}(t), \ldots, \rho_{N-1}^{\text{in}}(t)\right\} \tag{3}$$

$$\rho^{\text{out}}(t) = \left\{\rho_0^{\text{out}}(t), \ldots, \rho_{N-1}^{\text{out}}(t)\right\}, \tag{4}$$

for a video sequence $\rho = \left\{\rho^{\text{in}}, \rho^{\text{out}}\right\}$, around a sequence of centre points $\boldsymbol{c} = \{c(t)\}_{t=0,\ldots,T-1}$, where

$$\rho^{\text{in}} = \left\{\rho^{\text{in}}(0), \ldots, \rho^{\text{in}}(T-1)\right\} \tag{5}$$

$$\rho^{\text{out}} = \left\{\rho^{\text{out}}(0), \ldots, \rho^{\text{out}}(T-1)\right\}. \tag{6}$$

Our CRF model of radial values

We model the probability $P(\rho|\theta, D)$ modelled through a log-linear CRF

$$P(\rho|\theta, D) = \frac{1}{Z(\theta, D)} \exp(-E(\rho|\theta, D)). \tag{7}$$

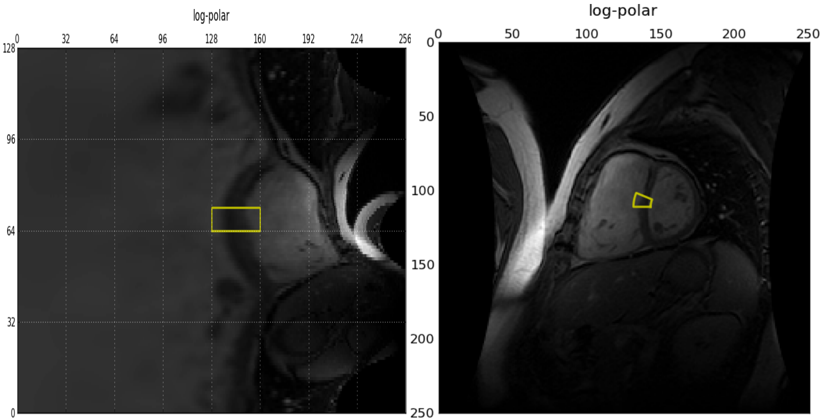
Energy is weighted sum of bivariate / 1st order feature functions relating local features of radii and image values over space and time.

$$E(\rho|\theta, D) = \sum_{q \in \mathcal{Q}} \theta_q f_q(\rho_q, D) \tag{8}$$

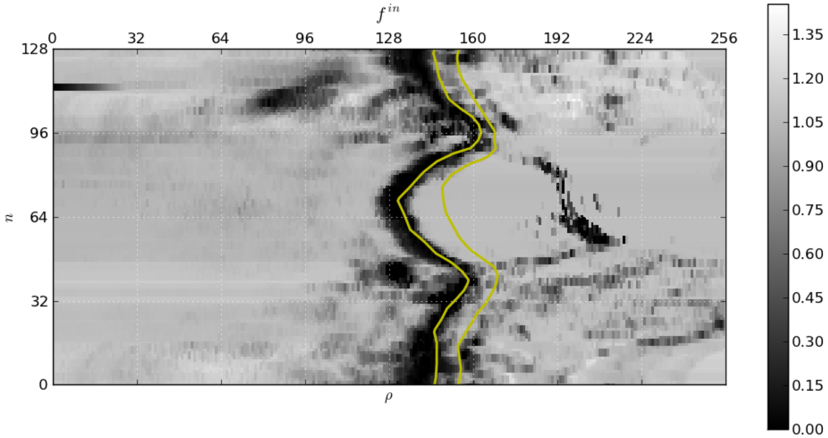
$$Z(\theta, D) = \sum_{\rho} \exp(-E(\rho|\theta, D)) \tag{9}$$

Feature function based on edge classifiers

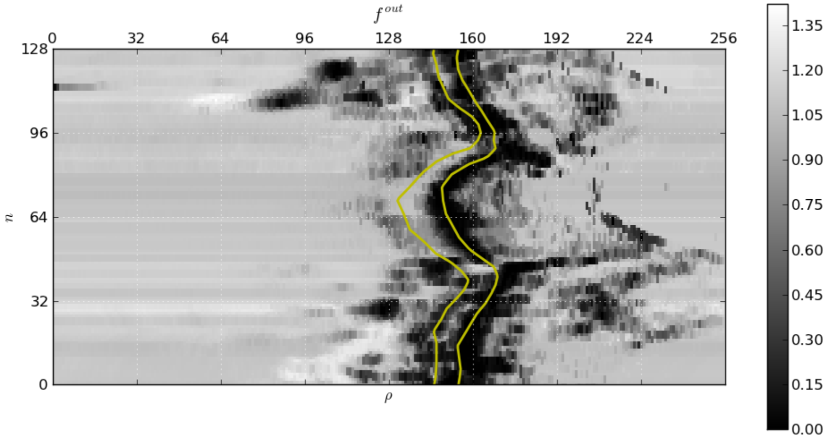
Sliding window, simple 2layer ANN with gradient info as input (trained on examples)



Inner contour cost function



Outer contour cost function



First order spatial and temporal variability

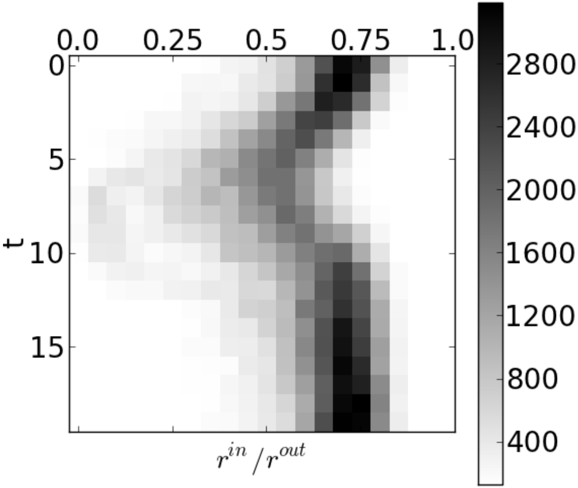
also wall colour consistency

$$f_r\left(\rho_n(t), \rho_{n-1}(t)\right) = \left(\frac{\rho_n(t) - \rho_{n-1}(t)}{M}\right)^2 \tag{10}$$

$$f_t\left(\rho_n(t), \rho_n(t-1)\right) = \left(\frac{\rho_n(t) - \rho_n(t-1)}{M}\right)^2 \tag{11}$$

$$f_t^{'}\left(\rho_n(t), \rho_n(t-1)\right) = \begin{cases} [\rho_n(t-1) < \rho_n(t)] & \text{if } t < t_{ES} \\ [\rho_n(t) < \rho_n(t-1)] & \text{otherwise.} \end{cases} \tag{12}$$

Histogram of inner/outer contours over time



Intensity variance of cardiac wall

variance of cardiac wall intensity between inner and outer

$$f_2^{\text{cross}}\left(\rho_n^{\text{in}}(t), \rho_n^{\text{out}}(t), \boldsymbol{d}_n(t)\right) = \frac{1}{W_n} \sum_{\rho=\rho_n^{\text{in}}(t)}^{\rho_n^{\text{out}}(t)} \left(\boldsymbol{d}_n(t,\rho) - \mu_n\right)^2 . \tag{13}$$

wall intensity consistency

$$f_t''\left(\rho_n^{\text{out}}(t), \rho_n^{\text{out}}(t-1)\right) = \left|\boldsymbol{d}_n\left(t, \rho_n^{\text{out}}(t) - \varepsilon_\rho\right) - \boldsymbol{d}_n\left(t-1, \rho_n^{\text{out}}(t-1) - \varepsilon\right.\right.$$

$$f_r''\left(\rho_n^{\text{out}}(t), \rho_{n-1}^{\text{out}}(t)\right) = \left|\boldsymbol{d}_n\left(t, \rho_n^{\text{out}}(t) - \varepsilon_\rho\right) - \boldsymbol{d}_n\left(t, \rho_{n-1}^{\text{out}}(t) - \varepsilon_\rho\right)\right| , \tag{15}$$

Weighted sum of feature functions

Combine these local features into energy

$$E\left(\rho|\theta, D\right) = \sum_{q \in \mathcal{Q}} \theta_q f_q\left(\rho_q, D\right). \tag{16}$$

Inference

Given a video D, find segmentation ρ

$$\rho^\star = \arg\max_\rho P(\rho|\theta, D). \tag{17}$$

since $Z(\theta, D)$ does not depend on the segmentation ρ

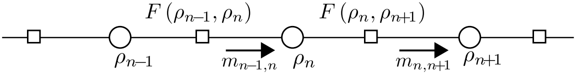
$$E(\rho|\theta, D) = \sum_{q\in\mathcal{Q}} \theta_q f_q(\rho_q, D). \tag{18}$$

$$\rho^\star = \arg\min_\rho E(\rho|\theta, D), \tag{19}$$

Belief Propagation = distributive law

Energy can be solved exactly if feature functions dependencies form a chain (or single loop)

$$\min_{\rho} E(\rho) = \min_{\rho_{N-1}} \ldots \min_{\rho_0} \Bigg(F\left(\rho_{N-1}, \rho_{N-2}\right) + \ldots + F\left(\rho_1, \rho_0\right) \Bigg), \quad (20)$$



Applying the distributive law

$$\min_{\rho} E(\rho) = \min_{\rho_{N-1}} \ldots \min_{\rho_1} \Bigg(F\left(\rho_{N-1}, \rho_{N-2}\right) + \ldots + F\left(\rho_2, \rho_1\right) + \underline{\min_{\rho_0} F\left(\rho_1, \rho_0\right)} \Bigg). \quad (21)$$

Belief Propagation = distributive law

Viterbi / dynamic programming / generalized distributive law

Repeating the process for the variables $\rho_1 \ldots \rho_{N-2}$, the following recursive expression is obtained

$$\min_{\rho} E(\rho) = \min_{\rho_{N-1}} \Big(m_{N-2 \to N-1} \left(\rho_{N-1} \right) \Big) \tag{22}$$

the "message" to ρ_{n+1} is defined as

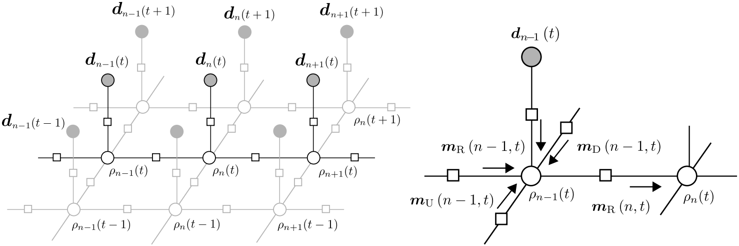
$$m_{n \to n+1} \left(\rho_{n+1} \right) = \min_{\rho_n} \Big(F \left(\rho_n, \rho_{n+1} \right) + m_{n-1 \to n} \left(\rho_n \right) \Big) \tag{23}$$

Starting with the calculation of $m_{0,1} \left(\rho_1 \right)$ and using (or *propagating*) its result to calculate $m_{1,2} \left(\rho_2 \right)$, etc., the minimum over all values of ρ requires only $O \left(NM^2 \right)$ operations

"Beam search" requires $O \left(NM\varepsilon_M \right)$

Loopy Belief Propagation

Graph of dependencies has many loops



Do it anyway. LBP = approximate, but well behaved.

Maximum likelihood estimation

$$\theta^{\star} = \arg\max_{\theta} \prod_{i} P\left(\rho^{(i)} | D^{(i)}, \theta\right), \tag{24}$$

$$\frac{\partial Z\left(\theta, D^{(i)}\right)}{\partial \theta_q} = -\sum_{\rho}\left(\exp\left(-\sum_{q'} \theta_{q'} f_{q'}\left(\rho_{q'}, D^{(i)}\right)\right) \cdot f_q\left(\rho_q, D^{(i)}\right)\right). \tag{25}$$

Ppartition (and derivative) sums over all configurations of ρ which requires $O\left(M^{2NT}\right)$

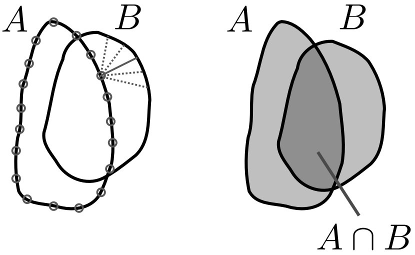
Quickly becomes intractable ($M = 256,\ N = 128,\ T = 20$)

Could approximate Z : *pseudolikelihood*

Avoid calculating $Z(\theta)$

Fundamentally, we are interested in obtaining the parameters $\theta^\star$ that would lead to a segmentation, $\rho^{\star(i)}$, of the sequence, that does not significantly differ from the ground truth annotated segmentation, $\rho^{(i)}$.

$$J(\theta) = \sum_i \text{error}\left(\rho^{(i)}, \rho^{\star(i)}\right). \tag{26}$$



Optimization

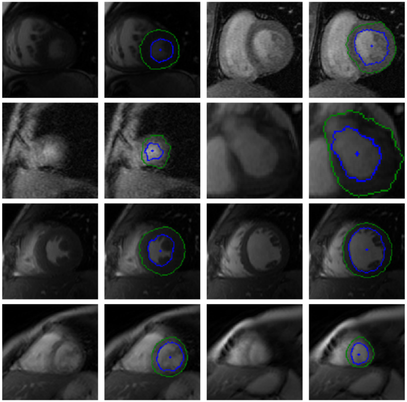
BFGS & numerically estimate gradient $\frac{\partial J'(\boldsymbol{w})}{\partial w_d} \approx \frac{J'(\boldsymbol{w}+\Delta\boldsymbol{w}_d)-J'(\boldsymbol{w})}{\|\Delta\boldsymbol{w}_d\|}$

- gradient estimate requires testing in each direction
- quickly reaches "OK" values, but does not improve further
- $\Delta\boldsymbol{w}_d$ problematic (too small=thinks it converged, too big=thinks it converged)

Powell's conjugate direction method

- line searches in changing base directions (avoid "zigzagging" towards optimum)
- improved results

Selected images from York dataset



Sunnybrook dataset

Results are comparable or outperforms the existing techniques (esp. inner contour)

Authors	Dice similarity		APD (mm)	
	inner	outer	inner	outer
Our method (trained on York)	0.87	0.92	2.70	2.23
Our method (after retraining)	**0.91**	**0.93**	**1.84**	**1.95**
Marak et al.	0.86	0.93	2.6	3.0
Lu et al.	0.89	0.94	2.07	1.91
Wijnhout et al.	0.89	0.93	2.29	2.28
Casta et al.	-	0.93	-	2.72
O'Brien et al.	0.81	0.91	3.73	3.16
Constantinides et al.	0.89	0.92	2.35	2.04
Huang S. et al.	0.89	0.94	2.10	1.95
Jolly	0.88	0.93	2.44	2.05

Summary

- Formulate problem as radial values around sequence of centre points
- Describe features correlating local properties of images and radial values
- Combined into a Conditional Random Field
- Inference and training (avoid partition function)
- Results are comparable or outperforms the existing techniques